

Quantum Theory of Ferromagnetism

Ferromagnetic substances are those substances which possess a spontaneous magnetic moment i.e. a magnetic moment even in the absence of an applied magnetic field. The Curie point T_c is the temperature at and above which the spontaneous magnetisation vanishes.

Weiss gave the theory of ferromagnetism on the basis of two hypotheses —

1) A ferromagnetic specimen contains in general a large number of small regions called domains which are spontaneously magnetised. The spontaneous magnetisation of the specimen is determined by the vector sum of the magnetic moments of the individual domains.

2) Within each domain the spontaneous magnetisation is due to existence of molecular field which tends to produce a parallel alignment of the atomic dipoles.

Existence of molecular moment leads to co-ordination or interaction between the atomic dipoles. Tendency of co-ordination is to produce parallel alignment.

To explain this fact Weiss assumed that magnetic field actually acting on the specimen is

$$H_{\text{eff}} = H + AI \quad \text{--- (1)}$$

On these assumptions we shall develop the theory.

Atom with quantum number J placed in magnetic field can set itself in the direction for which $\cos \theta = M_J / J$.

Component of Magnetic moment provided by atom under consideration = $M_J g \mu_B$.

Magnetic moment parallel to field $H = M_J g \mu_B H$.

Number of particles with a given value of M_J is proportional to $\exp(-E/kT)$ i.e. proportional to $\exp((M_J g \mu_B H)/kT)$.

Supposing that the ferromagnetic specimen contains N atoms/vol... the magnetisation of this volume will be given by the expression —

$$M = N \cdot \frac{\sum_{M_J=-J}^J M_J g \mu_B \cdot \exp(M_J g \mu_B H / kT)}{\sum_{M_J=-J}^J \exp(M_J g \mu_B H / kT)} \quad \text{--- (2)}$$

Substituting $(g \mu_B / kT) H = x$ in the above

equation, we get

$$M = Nq\mu_B \cdot \frac{\sum_{M_J=-J}^J M_J \exp M_J x}{\sum_{M_J=-J}^J \exp M_J x} \quad - (3)$$

$$= Nq\mu_B \cdot \frac{d}{dx} \left\{ \log \sum_{M_J=-J}^J \exp (M_J x) \right\} \quad - (4)$$

$$= Nq\mu_B \cdot \frac{d}{dx} \left\{ \log e (e^{Jx} + e^{(J-1)x} + \dots + e^{-Jx}) \right\}$$

$$M = Nq\mu_B \cdot \frac{d}{dx} \left\{ \log e^{Jx} (1 + e^{-x} + e^{-2x} + \dots + e^{-2Jx}) \right\}$$

$$= Nq\mu_B \cdot \frac{d}{dx} \left\{ \log e^{Jx} \cdot \left(\frac{1 - e^{-(2J+1)x}}{1 - e^{-x}} \right) \right\}$$

$$= Nq\mu_B \cdot \frac{d}{dx} \left\{ \log \frac{e^{Jx} - e^{-Jx} e^{-x}}{1 - e^{-x}} \right\} \quad - (5)$$

Multiplying and dividing eqn. (5) by $e^{x/2}$, we get

$$M = Nq\mu_B \cdot \frac{d}{dx} \left\{ \log \frac{e^{(J+1/2)x} - e^{-(J+1/2)x}}{e^{x/2} - e^{-x/2}} \right\}$$

$$= Nq\mu_B \cdot \frac{d}{dx} \left\{ \log \frac{\sinh (J + 1/2) x}{\sinh x/2} \right\}$$

$$= Nq\mu_B \cdot \frac{d}{dx} \left\{ \log \sinh (J + \frac{1}{2}) x - \log \sinh \frac{x}{2} \right\}$$

$$= Nq\mu_B \cdot \left[\frac{2J+1}{2} \coth \frac{2J+1}{2} x - \frac{1}{2} \coth \frac{x}{2} \right] \quad - (6)$$

Substituting $x = a/J$ in eqn. (6), we get

$$M = N g \mu_B \cdot \left[\frac{2J+1}{2} \coth \frac{2J+1}{2J} a - \frac{1}{2} \coth \frac{a}{2J} \right] \quad \text{--- (7)}$$

$$= N J g \mu_B \cdot \left[\frac{2J+1}{2J} \coth \frac{2J+1}{2J} a - \frac{1}{2J} \coth \frac{a}{2J} \right]$$

$$\text{or } M = N J \mu_B B(a) \quad \text{--- (8)}$$

where $B(a)$ is the Brillouin function.

From the two substitutions made in the above treatment we have,

$$\frac{a}{J} = \frac{g \mu_B H}{kT}$$

$$\text{or } a = \frac{J g \mu_B H}{kT} \quad \text{--- (9)}$$

But in the case of ferromagnetism H is replaced by H_{eff} given by the relation

$$H_{\text{eff}} = H + A I$$

$$\text{So } a = \frac{J g \mu_B H_{\text{eff}}}{kT} = \frac{J g \mu_B (H + A I)}{kT}$$

For spontaneous magnetisation $H = 0$

$$\therefore a = \frac{J g \mu_B A I}{kT} \quad \text{or } I = \frac{a kT}{J g \mu_B A} = M \quad \text{--- (10)}$$

[since $I = (M/v)$ and we have derive this per unit volume].

since M must satisfy equation (8) and (10), its value at a given temperature may be obtained from the point of intersection S of the two corresponding M values.

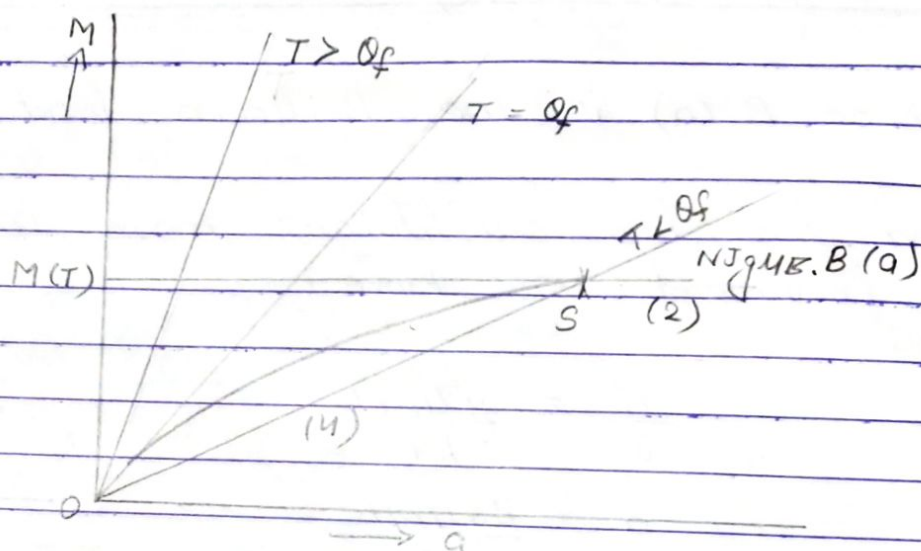


Fig. Schematic representation of the method for finding the spontaneous magnetisation of temperature T . The point of intersection S determines $M(T)$.

Equation (10) represents a straight line; the slope of the line is proportional to temperature. From figure 9 it follows that for $T < T_c$, M value although $H = 0$. Further, since the origin 0 is also a point of intersection; it can be shown that free energy of the state with non-vanishing M

value is smaller than that for $M=0$, because the greater is the free energy the lesser is M , i.e., the point of intersection O is unstable. Hence for $T < \theta_f$ the spontaneous magnetisation results.

For $T = \theta_f$ the slope of the straight line curve represented by eqn. (10) is equal to that of the tangent of curve (2) at the origin. Thus for $T > \theta_f$ the two curves don't intersect and hence there is no spontaneous magnetisation.

It will be evident that there must exist a relation between the curie point θ_f and molecular field constant A .

In fact one expects θ_f increase with A . because the tendency of parallel alignment increases as A become larger. In order to establish this relation we make use of the fact that $a \ll 1$ (near the origin in figure) the Brillouin function is given by

$$B(a) \approx \frac{J+1}{3J} a$$

Hence the tangent of the curve (2) has the slope equal to $Ng\mu_B [(J+1)/3]$.

We get it by putting $B(a) = [J+1]/3J$ a
in expression (8).

Thus

$$M = N J g \mu_B \cdot B(a) = N J g \mu_B \cdot \frac{J+1}{3J} a$$
$$= N g \mu_B \frac{J+1}{3} a \quad \text{--- (11)}$$

Hence slope is $N g \mu_B \cdot [(J+1)/3]$

Putting this slope equal to the slope of
curve given by eqn. (10),
for $T = \theta_f$.

$$\frac{k \theta_f}{J g \mu_B A} = N g \mu_B \cdot \frac{J+1}{3}$$

$$\text{or } \frac{3k \theta_f}{A} = N g^2 \mu_B^2 \cdot J(J+1) \quad \text{--- (12)}$$

Hence θ_f is proportional to molecular field
constant A .